Large Deviations of Shepp Statistics for Fractional Brownian Motion

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June 12, 2013

Abstract: Define the incremental fractional Brownian field $Z_H(\tau, s) = B_H(s+\tau) - B_H(s), H \in (0,1)$, where $B_H(s)$ is a standard fractional Brownian motion with Hurst index $H \in (0,1)$. In this paper we derive the exact asymptotic behaviour of the maximum $M_H(T) = \max_{(\tau,s)\in[0,1]\times[0,T]} Z_H(\tau, s)$ for any $H \in (0,1/2)$ complimenting thus the result of Zholud (2008) which establishes the exact tail asymptotic behaviour of $M_{1/2}(T)$.

Key Words: Shepp statistics; scan statistics; exact asymptotics; fractional Brownian motion.

AMS Classification: Primary 60G15; secondary 60G70

1 Introduction

Let $\{B_H(t), t \geq 0\}$ be a standard fractional Brownian motion (fBm) with Hurst index $H \in (0,1)$ which is a centered H-self-similar Gaussian process with stationary increments, almost surely continuous sample paths, $B_H(0) = 0$ and its covariance function is given by

$$Cov(B_H(t), B_H(s)) = \frac{1}{2}(t^{2H} + s^{2H} - \mid t-s \mid^{2H}), \quad t, s \geq 0.$$

An important random field defined in terms of this fBm is the so-called incremental fractional Brownian motion

$$Z_H(\tau, s) = B_H(s+\tau) - B_H(s), \quad s, \tau \geq 0.$$

In various statistical applications the incremental fractional Brownian motion appears as the limit model. Typically, when independent and identical observations are modeled, then the limit model has $H = 1/2$. A closely related random field, namely the standardised incremental fractional Brownian motion

$$Z^*_H(\tau, s) = \frac{B_H(s+\tau) - B_H(s)}{\tau^H}, \quad s, \tau \in (0, \infty)$$

serves also in various applications as a limit model. For instance with motivation from queuing theory, consider $\{K(t), t \geq 0\}$ a homogeneous Poisson process with $\mathbb{E}\{K(t)\} = \lambda t, \lambda > 0$ and set for some T, τ positive $K(\tau, T) := \sup_{0\leq s \leq T}(K(s+\tau) - K(s))$. The random variable $K(\tau, T)$ is the maximum service length of an $M/G/\infty$ queue with deterministic service time τ; it is also the version of scan statistics on the positive half line (see e.g., Cressie (1980)). For the study of $K(\tau, T)$ the following convergence in distribution

$$\frac{K(\tau, T) - \lambda\tau}{\sqrt{\lambda\tau}} \to \sup_{0\leq s\leq T} Z^*_{1/2}(\tau, s), \quad \lambda \to \infty$$

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is important since the distribution function of $\sup_{0\le s\le T} Z^*_{1/2}(\tau,s)$ is derived in Shepp (1971), see also Slepian (1961), Shepp (1966) and Theorem 3.2 in Cressie (1980).

Various authors refer to the process $\{Z^*_H(\tau,T),\tau\ge 0\}$ as the standardised Shepp statistics. Important results for Shepp statistics and related quantities can be found in Deheuvels and Devroye (1987), Siegmund and Venkatraman (1995), Dümbgen and Spokoiny (2001), Kabluchko and Munk (2008) and Zholud (2009).

The recent papers Zholud (2008) and Kabluchko (2007,2011a) present asymptotic results on the extremes of Shepp statistics and standardised Shepp statistics, i.e., therein the tail asymptotic behaviour of

$$M_H(T)=\sup_{\tau\in[0,1],0\le s\le T} Z_H(\tau,s) \quad \text{and } M^*_H(a,b,T)=\sup_{\tau\in[a,b],0\le s\le T} Z^*_H(\tau,s), \quad 0<a<b<\infty$$

for the case $H=1/2$ is investigated dealing thus with the increments of the Brownian motion.

In view of Zholud (2008) for any $T>0$

$$P\left(M_{1/2}(T)>u\right)=\widetilde{\mathcal{H}}_* T u^2\Psi(u)(1+o(1)), \quad u\to\infty, \tag{1}$$

where Ψ is the survival function of a $N(0,1)$ random variable and the constant $\widetilde{\mathcal{H}}_*$ is given by

$$\widetilde{\mathcal{H}}_*=\lim_{a\to\infty}\lim_{b\to\infty} a^{-1}e^{-\frac{a+b}{2}}\mathbb{E}\Big\{\exp\Big(\max_{\substack{0\le t\le a\\ 0\le s\le b}} B_{1/2}(t+s+a)-B_{1/2}(t)\Big)\Big\}.$$

For any Hurst index $H\neq 1/2$ the independence of the increments of B_H does not hold, which has been the crucial property in the derivation of (1). In our main result given in Theorem 2.1 we derive the exact asymptotics of $M_H(T)$ for $H\in(0,1/2)$, which is the well-known short-range dependence case for fBm. If $H\in(1/2,1)$, thus we have a long-range dependence, we have a much more involved problem which will be therefore considered elsewhere.

Numerous authors have considered properties and characterisations of fBm, see e.g., Mishura and Valkeila (2011), Kabluchko (2011b) and the references therein. Our contribution present a new result for the Shepp statistics of fBm, which we believe is important for both future theoretical and applied developments.

Clearly, our result on the tail asymptotic behaviour of $M_H(T)$ implies certain asymptotic bounds for the tail asymptotics of $M^*_H(a,b,T)$. The exact asymptotics of $M^*_H(a,b,T)$ is however easier to deal with and follows by a direct application of the results of Chan and Lai (2006) since the pertaining random field is locally stationary; see Mikhaleva and Piterbarg (1996), and Piterbarg (1996) for the main findings concerning locally stationary random fields.

Brief outline of the rest of the paper: Section 2 displays the main result, its proof is given in Section 3.

2 Main Result

In the asymptotic theory of Gaussian processes two important constants are crucial, namely the Pickands and Piterbarg constants. Since in our results only the former constant appears, we briefly mention that it is defined by

$$\mathcal{H}_{2H}=\lim_{\lambda\to\infty}\lambda^{-1}\mathbb{E}\left\{\exp\left(\max_{t\in[0,\lambda]}\left(\sqrt{2}B_H(t)-t^{2H}\right)\right)\right\}\in(0,\infty).$$

It is well-known that $\mathcal{H}_1 = 1$ and $\mathcal{H}_2 = 1/\sqrt{\pi}$; see Piterbarg (1972) which gives the first rigorous proof of Pickands theorem presented in Pickands (1969), Dębicki (2002), Wu (2007), Dębicki and Kisowski (2009) and Dębicki and Tabiś (2011) for generalisations of Pickands constant.

The result of (1) is of some importance for dealing with the general case $H \neq 1/2$. However we cannot use the method of proof in Zholud (2008) which relies on the independence of increments of Brownian motion. Our proof of the main result presented below is strongly motivated by the method utilised in the seminal contribution Piterbarg (2001).

Theorem 2.1. *For any $H \in (0, 1/2)$ and any $T > 0$*

$$P\left(M_H(T) > u\right) = \frac{T}{H}\left(\frac{1}{2}\right)^{1/H} \mathcal{H}_{2H}^2 u^{\frac{2}{H}-2}\Psi(u)(1+o(1)) \tag{2}$$

holds as $u \to \infty$.

Remark. a) As in Zholud (2008) also for the case $H \in (0, 1/2)$ it is possible to derive a similar expansion as in (2) when $T = T_u$ depends on u. From our proof we see that the result holds if $T = T_u$ is such that $\lim_{u\to\infty} T_u u^{1/d} = \infty$ for some $d \in (H, 1/2)$. Following word-by-word the arguments of Hüsler and Piterbarg (2004) the proof of the case $T_u < \exp(cu^2)$ can also be included when $c \in (0, 1/2)$.

b) As in Hüsler and Piterbarg (2004), we obtain that for $H \in (0, 1/2)$ the Gumbel limit law

$$\lim_{T\to\infty} \max_{x\in\mathbb{R}} \Big| P\left(a_T(M_H(T) - b_T) \le x\right) - \exp(-e^{-x})\Big| = 0, \tag{3}$$

holds, where

$$a_T = \sqrt{2\ln T}, \quad b_T = a_T + a_T^{-1}\Big[\left(\frac{1}{H} - \frac{3}{2}\right)\ln\ln T + \ln(2^{-3/2}\mathcal{H}_{2H}^2 H^{-1}(2\pi)^{-1/2})\Big]. \tag{4}$$

c) The tail asymptotics of $M_H^*(a, b, T)$ and the Gumbel limit law follow by utilising the result of Chan and Lai (2006) and Piterbarg (2001) since the pertaining random field is locally stationary; see Mikhaleva and Piterbarg (1996), and Piterbarg (1996) for the main findings concerning locally stationary random fields.

3 Proofs

We present first a lemma which is crucial for the proof of Theorem 2.1.

Lemma 3.1. *Let $\varepsilon \in (0, H), H \in (0, 1)$ and let $T > 0$ be given. Then for all large u and $\delta_u := \ln^2 u/u^2$*

$$P\left(\max_{\substack{\tau\in[0,1-\delta_u]\\ s\in[0,T]}} Z_H(\tau, s) > u\right) \le CTu^{2/H-1}\exp\left(-\frac{1}{2}u^2 - (H-\varepsilon)\ln^2 u\right) \tag{5}$$

holds with $C > 0$ not depending on T and u.

Proof: For any τ, s, τ', s' positive we have

$$\mathbb{E}\Big\{(Z_H(\tau, s) - Z_H(\tau', s'))^2\Big\} \le G[|\tau - \tau'|^{2H} + |s - s'|^{2H}],$$

with some constant $G > 0$. Consequently, by Piterbarg inequality (given in Theorem 8.1 in Piterbarg (1996), see also Theorem 8.1 in Piterbarg (2001) and Proposition 3.2 in Tan and Hashorva (2013)) for any $x \in (0,1)$ we have

$$P\left(\max_{\substack{\tau\in[0,x]\\ s\in[0,T]}} Z_H(\tau,s) > u\right) \le CTu^{2/H-1}\exp\left(-\frac{u^2}{2x^{2H}}\right)$$

for some C independent of x and u. For $x = 1-\ln^2 u/u^2$ we obtain

$$P\left(\max_{\substack{\tau\in[0,x]\\ s\in[0,T]}} Z_H(\tau,s) > u\right) \quad \le \quad CTu^{2/H-1}\exp\left(-\frac{u^2}{2(1-\ln^2 u/u^2)^{2H}}\right),$$

hence the proof follows. □

Proof of Theorem 2.1. Let in the following $R_u = u^{-2/H'}$ for some $H' \in (2H,1)$ and let $\delta_u = u^{-2}\ln^2 u$. By Lemma 3.1, we can restrict the considered domain of (τ,s) to $\tau \in [1-\delta_u, 1]$. For this choice of R_u, we have

$$\lim_{u\to\infty}\frac{\delta_u}{R_u} = \infty, \qquad \text{and } \lim_{u\to\infty} u^2 R_u = 0. \tag{6}$$

Note that with $\mathcal{A} := \{(t,s)\in\mathbb{R}^2 : t\in[0,T+1], s\in[0,T], t-s\in[0,1]\}$

$$M_H(T) = \max_{(t,s)\in\mathcal{A}} Y_H(t,s), \qquad \text{where } Y_H(t,s) = B_H(t) - B_H(s).$$

Define next for any $l = 1,\cdots,[TR_u^{-1}]$

$$J_{k,l} = [1+(l-k)R_u, 1+(l-k+1)R_u]\times[(l-1)R_u, lR_u],$$

with $k = 1,\cdots,[\delta_u R_u^{-1}+1]$ and

$$J'_{k,l} = [1+(l-1-k)R_u, 1+(l-k)R_u]\times[(l-1)R_u, lR_u],$$

where $k = 1,\cdots,[\delta_u R_u^{-1}-1]$.

The variance function $\sigma^2(\tau,s)$ of $Z_H(\tau,s)$ equals τ^{2H}. Consequently, $\sigma(1,s) = 1$ for all $s \in [0,1]$ and hence the maximum point of the variance is not a single point but taken on $(1,s), s \in (0,1)$. Taylor expansion yields

$$\sigma(\tau,s) = \sigma_Z(\tau) = 1 - H(1-\tau) + o(1-\tau)^2$$

as $\tau \uparrow 1$. Hence, for arbitrarily small $\varepsilon > 0$ and for all sufficiently large u, on $J_{k,l}$ the variance $\sigma_Y^2(t,s)$ of $Y_H(t,s)$ satisfies

$$\sigma_Y(t,s) \le 1 - (H-\varepsilon)(k-2)R_u.$$

On $J'_{k,l}$ the variance $\sigma_Y^2(t,s)$ of $Y_H(t,s)$ satisfies

$$1 - (H+\varepsilon)(k+1)R_u \le \sigma_Y(t,s).$$

For the correlation function $r_Y(t,s;t',s')$ of $Y_H(t,s)$, we have

$$r_Y(t,s;t',s') = 1 - \frac{1}{2}(1+o(1))(|t-t'|^{2H} + |s-s'|^{2H})$$

as $t-s, t'-s' \uparrow 1$, $s-s' \to 0$ and $t-t' \to 0$. Consider the centered and homogeneous Gaussian random field $\zeta_{H\pm\varepsilon}(t,s)$ with covariance function

$$\exp\left(-(\frac{1}{2}\pm\varepsilon)(|s-s'|^{2H}+|t-t'|^{2H})\right), \quad s,s',t,t' \in \mathbb{R},$$

where $\varepsilon \in (0,H)$. From this point, we assume below that $H \in (0,1/2)$. Using Slepian inequality and Theorem 7.2. of Piterbarg (1996) for all sufficiently large u we have

$$P\left(\max_{(t,s)\in J_{k,l}} Y_H(t,s) > u\right) \le (\frac{1}{2}+\varepsilon)^{\frac{1}{H}} \mathcal{H}_{2H}^2 u_{k-}^{\frac{2}{H}} R_u^2 \Psi(u_{k-}) =: \eta(\varepsilon, u_{k-}) \tag{7}$$

and

$$P\left(\max_{(t,s)\in J'_{k,l}} Y_H(t,s) > u\right) \ge (\frac{1}{2}-\varepsilon)^{\frac{1}{H}} \mathcal{H}_{2H}^2 u_{k+}^{\frac{2}{H}} R_u^2 \Psi(u_{k+}) =: \eta(-\varepsilon, u_{k+}), \tag{8}$$

where

$$u_{k-} = \frac{u}{1-(H-\varepsilon)(k-2)R_u}, \quad u_{k+} = \frac{u}{1-(H+\varepsilon)(k+1)R_u}.$$

Further, for all sufficiently large u we obtain utilising further (6) (set $H_\varepsilon := H-\varepsilon, T_\varepsilon := (1/2+\varepsilon)^{\frac{1}{H}} T\mathcal{H}_{2H}^2$)

$$\begin{aligned}
P\left(\max_{(t,s)\in\mathcal{A}} Y_H(t,s) > u\right) &\le P\left(\max_{(t,s)\in\cup J_{k,l}} Y_H(t,s) > u\right) \\
&\le \sum_{k=1}^{[R_u^{-1}\delta_u+1]} \sum_{l=1}^{[TR_u^{-1}]+1} \eta(\varepsilon, u_{k-}) \\
&= ([TR_u^{-1}]+1) \sum_{k=1}^{[R_u^{-1}\delta_u+1]} \eta(\varepsilon, u_{k-}) \\
&= T_\varepsilon \frac{1+o(1)}{\sqrt{2\pi}} u^{\frac{2}{H}-1} e^{-\frac{u^2}{2}} \sum_{k=1}^{[R_u^{-1}\delta_u+1]} \frac{e^{-\frac{1}{2}(u_{k-}^2-u^2)} R_u}{(1-H_\varepsilon(k-2)R_u)^{\frac{2}{H}-1}} \\
&\le \frac{T_\varepsilon}{1-\varepsilon'} u^{\frac{2}{H}} \Psi(u) \sum_{k=1}^{[R_u^{-1}\delta_u]+2} \exp\Big(\frac{-u^2}{2}\Big(\frac{1}{(1-H_\varepsilon(k-2)R_u)^2}-1\Big)\Big) R_u \\
&\le \frac{T_\varepsilon}{1-\varepsilon'} u^{\frac{2}{H}-2} \Psi(u) \sum_{k=1}^{[R_u^{-1}\delta_u]+2} \exp\Big(-\frac{H_\varepsilon}{1+\varepsilon'''}(k-2)u^2 R_u\Big) u^2 R_u \\
&= \frac{T_\varepsilon}{1-\varepsilon'} u^{\frac{2}{H}-2} \Psi(u) \int_0^\infty \exp\Big(-\frac{H_\varepsilon}{1+\varepsilon'''}x\Big)\, dx (1+o(1)) \\
&= \frac{T_\varepsilon(1+\varepsilon''')}{H_\varepsilon(1-\varepsilon')} u^{\frac{2}{H}-2} \Psi(u)(1+o(1)), \quad u \to \infty,
\end{aligned} \tag{9}$$

where $\varepsilon', \varepsilon'', \varepsilon''' \in (0,1)$ above are appropriately chosen constants and the passing from the sum to the integral is legitime since (6) holds. Similarly, for all sufficiently large u

$$\begin{aligned}
&P\left(\max_{(t,s)\in\mathcal{A}} Y_H(t,s) > u\right) \\
&\ge P\left(\max_{(t,s)\in\cup J'_{k,l}} Y_H(t,s) > u\right) \\
&\ge \sum_{l=1}^{[TR_u^{-1}]} \sum_{k=1}^{[R_u^{-1}\delta_u-1]} \eta(-\varepsilon, u_{k+}) - \sum_{(l,k),(l',k'):\rho((s,t),(s',t'))\ge R_u} P\left(\max_{(t,s)\in J'_{k,l}} Y_H(t,s) > u, \max_{(t',s')\in J'_{k',l'}} Y_H(t',s') > u\right)
\end{aligned}$$

$$
- \sum_{\substack{(l,k),(l',k'):\rho((s,t),(s',t'))<R_u \\ (l,k)\neq(l',k')}} P\left(\max_{(t,s)\in J'_{k,l}} Y_H(t,s) > u, \max_{(t',s')\in J'_{k',l'}} Y_H(t',s') > u \right), \tag{10}
$$

where $\rho(\cdot,\cdot)$ is the Euclidean distance of two points in $\mathbb{R}^2$. The first sum can be bounded from below with the same arguments as in the proof of (9) by

$$
(1/2-\varepsilon)^{\frac{1}{H}} \frac{T}{H+\varepsilon} \mathcal{H}_{2H}^2 u^{\frac{2}{H}-2} \Psi(u)(1+o(1)).
$$

A generic term of the second sum can be estimated as follows. The Gaussian field $W_H(t,s,t',s') = Y_H(t,s)+Y_H(t',s')$ on $J'_{k,l} \times J'_{k',l'}$ has for $H \in (0,1/2)$ variance function

$$
2+2r_Y(t,s;t',s') \quad = \quad 4-(|t-t'|^{2H}+|s-s'|^{2H})(1+o(1)) \le 4-(2-\varepsilon)R_u^{2H},
$$

where the inequality holds for all large u and $\varepsilon \in (0,H)$. Consequently, for some $C>0, c_*>0$ by Piterbarg inequality

$$
\begin{aligned}
P\left(\max_{(t,s)\in J'_{k,l}} Y_H(t,s) > u, \max_{(t',s')\in J'_{k',l'}} Y_H(t',s') > u \right) &\le P\left(\max_{(t,s,t',s')\in J'_{k,l}\times J'_{k',l'}} W_H(t,s,t',s') > 2u \right) \\
&\le C R_u^2 u^{\frac{4}{H}} \Psi\left(\frac{u}{\sqrt{1-(1/2-\varepsilon)R_u^{2H}}} \right) \\
&= C R_u^2 u^{\frac{4}{H}} \exp\left(-c_* u^2 R_u^{2H}\right) \Psi(u) \\
&= o(u^a \Psi(u)), \quad u \to \infty
\end{aligned}
$$

for any real a, since $H<1/2$ and $H' \in (2H,1)$ imply

$$
u^2 R_u^{2H} = u^{2(1-2H/H')} \to \infty, \quad u \to \infty.
$$

Therefore, in order to complete the proof we need to consider only the third term on the right-hand side of (10). The generic term in this sum is equal to

$$
\begin{aligned}
& P\left(\max_{(t,s)\in J'_{k,l}} Y_H(t,s) > u \right) + P\left(\max_{(t,s)\in J'_{k',l'}} Y_H(t,s) > u \right) - P\left(\max_{(t,s)\in J'_{k,l}\cup J'_{k',l'}} Y_H(t,s) > u \right) \\
&= 2P\left(\max_{(t,s)\in J'_{k,l}} Y_H(t,s) > u \right) - P\left(\max_{(t,s)\in J'_{k,l}\cup J'_{k',l'}} Y_H(t,s) > u \right).
\end{aligned} \tag{11}
$$

The first term on the right-hand side of (11) has been already estimated. For the second one note that $\rho(s,t;s',t') < R_u$, so both $l-l'$ and $k-k'$ cannot be greater than 2. The variance of the random field $\{Y_H(t,s),\ (t,s)\in J'_{k,l}\cup J'_{k',l'}\}$, satisfies

$$
1-(H+\varepsilon)(k+1)R_u \le \sigma_Y(t,s) \le 1-(H-\varepsilon)(k-2)R_u.
$$

Hence we use (7) and (8) with the same definition of $u_{k\pm}$. Consequently, summing (11) we obtain a sum which tends to an integral and which terms are uniformly negligible with respect to the corresponding terms in the first sum in the right-hand side of (10). Finally, letting $\varepsilon+\varepsilon'+\varepsilon''+\varepsilon''' \downarrow 0$ establishes the proof. $\square$

Acknowledgement: We are thankful to the referees of the paper as well as to Krzysztof Dębicki, Lanpeng Ji, Yuliya Mishura and Yimin Xiao for numerous suggestions which improved this paper significantly. E. Hashorva kindly acknowledge support by the Swiss National Science Foundation Grants 200021-1401633/1 and 200021-134785 as well as by the project RARE -318984, a Marie Curie International Research Staff Exchange Scheme Fellowship within the 7th European Community Framework Programme. Tan's work was supported by National Science Foundation of China (No. 11071182) and Research Start-up Foundation of Jiaxing University (No. 70512021).

References

[1] Chan, H.P., Lai, T.L., 2006. Maxima of asymptotically Gaussian random fields and moderate deviation approximations to boundary crossing probabilities of sums of random variables with multidimensional indices. Ann. Probab., 34, 80-121.

[2] Cressie, N., 1980. The asymptotic distribution of scan statistic under uniformity. Ann. Probab., 80, 828-840.

[3] Dębicki, K., 2002. Ruin probability for Gaussian integrated processes. Stoch. Proc. Appl., 98, 151-174.

[4] Dębicki, K., Kisowski, P. 2009. A note on upper estimates for Pickands constants. Stat. Probab. Letters, 78, 2046-2051.

[5] Dębicki, K., Tabiś, K., 2011. Extremes of time-average stationary Gaussian processes. Stoch. Proc. Appl., 121, 2049-2063.

[6] Deheuvels, P., Devroye, L., 1987. Limit laws of Erdös-Rényi-Shepp type. Ann. Probab. 15, 1363-1386.

[7] Dümbgen, L., Spokoiny, V.G., 2001. Multiscale testing of qualitative hypotheses. Ann. Statist. 29, 124-152.

[8] Hüsler, J., Piterbarg, V.I., 2004. Limit theorem for maximum of the storage process with fractional Brownian motion as input. Stoch. Proc. Appl., 114, 231-250.

[9] Kabluchko, Z., 2007. Extreme-value analysis of standardized Gaussian increments. http://www.arxiv.org/abs/0706.1849.

[10] Kabluchko, Z., 2011a. Extremes of the standardized Gaussian noise. Stoch. Proc. Appl., 121, 515-533.

[11] Kabluchko, Z. 2011b. Extremes of independent Gaussian processes. Extremes, 14, 285–310.

[12] Kabluchko, Z., Munk, A., 2008. Exact convergence rate for the maximum of standardized Gaussian increments. Elect. Comm. in Probab., 13, 302-310.

[13] Mikhaleva, T.L., Piterbarg, V.I. 1996. On the distribution of the maximum of a Gaussian field with a constant variance on a smooth manifold. Theory Probab. Appl. 41, 367-379.

[14] Mishura, Y., Valkeila, E., 2011. An extension of the Lévy characterization to fractional Brownian motion. Ann. Probab., 39, 439-470.

[15] Pickands, J., III., 1969. Asymptotic properties of the maximum in a stationary Gaussian process. Trans. Am. Math. Soc., 145, 75-86.

[16] Piterbarg, V.I., 1972. On the paper by J. Pickands "Upcrosssing probabilities for stationary Gaussian processes". Vestnik Moscow. Univ. Ser. I Mat. Mekh. 27, 25-30. English translation of Moscow Univ. Math. Bull., 27.

[17] Piterbarg, V.I., Asymptotic Methods in the Theory of Gaussian Processes and Fields, AMS, Providence, R.I., 1996.

[18] Piterbarg, V.I., 2001. Large Deviations of a storage process with fractional Browanian motion as input. Extremes, 4, 147-164.

[19] Shepp, L.A., 1966. Radon-Nykodym derivatives of Gaussian measures. Ann. Math. Stat. 37, 321-354.

[20] Shepp, L.A., 1971. First passage time for a particular Gaussian process. Ann. Math. Statist., 42, 946-951.

[21] Siegmund, S., Venkatraman, E.S., 1995. Using the generalized likelihood ratio statistic for sequential detection of a change-point. Ann. Statist., 23, 255-271.

[22] Slepian, D., 1961. First passage time for a particular Gaussian process. Ann. Math. Statist., 32, 610-612.

[23] Tan, Z., Hashorva, E., 2013. Exact asymptotics and limit theorems for supremum of stationary chi-processes over a random interval. Stoch. Proc. Appl., 123, 2983–2998.

[24] Zholud, D., 2008. Extremes of Shepp statistics for the Wiener process. Extremes, 11, 339-351.

[25] Zholud, D., 2009. Extremes of Shepp statistics for Gaussian random walk. Extremes, 12, 1-17.

[26] Wu, D. 2007. Generalized Pickands constants. Journal of Mathematical Physics, 48, 053513-053513.